Leibniz Universität Hannover

11th Northern German Differential Geometry Day

Programme:

- 11:30-12:30: **Felix Schulze** (Warwick)
  Ancient solutions and translators in Lagrangian mean curvature flow
- Lunch
- 14:45-15:45: **Alexander Bobenko** (TU Berlin)
  The Bonnet problem: Is a surface characterized by its metric and curvatures?
- 16:45-17:45: **Thomas Walpuski** (HU Berlin)
  The Gopakumar–Vafa finiteness conjecture

July 15, 2022
Leibniz Universität Hannover

Seminar room 016, Callinstraße 23 (building 3110)

Organizers:
Vicente Cortés (Universität Hamburg)
Jens Heber, Hartmut Weiß (Christian-Albrechts-Universität zu Kiel)
Roger Bielawski, Lynn Heller, Sebastian Heller, Knut Smoczyk
(Leibniz Universität Hannover)

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- **Ancient solutions and translators in Lagrangian mean curvature flow** (Felix Schulze)

For almost calibrated Lagrangian mean curvature flow it is known that all singularities are of Type II. To understand the finer structure of the singularities forming, it is thus necessary to understand the structure of general ancient solutions arising as potential limit flows at such singularities. We will discuss recent progress showing that ancient solutions with a blow-down a pair of static planes meeting along a 1-dimensional line are translators. This is joint work with J. Lotay and G. Szekelyhidi.

- **The Bonnet problem: Is a surface characterized by its metric and curvatures?** (Alexander Bobenko)

We consider a classical problem in differential geometry, known as the Bonnet problem, whether a surface is characterized by a metric and mean curvature function. Generically, the answer is yes. Special cases when it is not the case are classified. In particular, we explicitly construct a pair of immersed tori that are related by a mean curvature preserving isometry. This resolves a longstanding open problem on whether the metric and mean curvature function determine a unique compact surface. Discrete differential geometry is used to find crucial geometric properties of surfaces. This is a joint work with Tim Hoffmann and Andrew Sageman-Furnas.

- **The Gopakumar–Vafa finiteness conjecture** (Thomas Walpuski)

The purpose of this talk is to illustrate an application of the powerful machinery of geometric measure theory to a conjecture in Gromov–Witten theory arising from physics. Very roughly speaking, the Gromov–Witten invariants of a symplectic manifold (X,ω) equipped with a tamed almost complex structure J are obtained by counting pseudo-holomorphic maps from mildly singular Riemann surfaces into (X,J). It turns out that Gromov–Witten invariants are quite complicated (or "have a rich internal structure"). This is true especially for if (X,ω) is a symplectic Calabi–Yau 3–fold (that is: $\dim X = 6$, $c_1(X,\omega) = 0$).
In 1998, using arguments from M–theory, Gopakumar and Vafa argued that there are integer BPS invariants of symplectic Calabi–Yau 3–folds. Unfortunately, they did not give a direct mathematical definition of their BPS invariants, but they predicted that they are related to the Gromov–Witten invariants by a transformation of the generating series. The Gopakumar–Vafa conjecture asserts that if one defines the BPS invariants indirectly through this procedure, then they satisfy an integrality and a (genus) finiteness condition.
The integrality conjecture has been resolved by Ionel and Parker. A key innovation of their proof is the introduction of the cluster formalism: an ingenious device to side-step questions regarding multiple covers and super-rigidity. Their argument could not resolve the finiteness conjecture, however. The reason for this is that it relies on Gromov's compactness theorem for pseudo-holomorphic maps which requires an a priori genus bound. It turns out, however, that Gromov's compactness theorem can (and should!) be replaced with the work of Federer–Flemming, Allard, and De Lellis–Spadaro–Spolaor. This upgrade of Ionel and Parker's cluster formalism proves both the integrality and finiteness conjecture.
This talk is based on joint work with Eleny Ionel and Aleksander Doan.